

MAT220 Final Exam Review

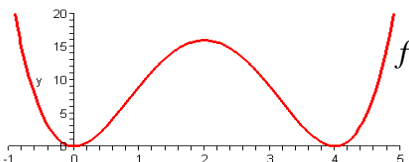
1. Let $f(x)$ be an even function and $g(x)$ be an odd function. Complete the following table.

x	$f(x)$	$g(x)$
-2	5	5
-1	4	4
0	0	0
1		
2		

2. Consider the functions f , g and h in the table below.

x	$f(x)$	$g(x)$	$h(x)$
0	10	10	10
1	9	7	5
2	6	4	2
3	1	1	1

- a. Are these functions **increasing** or **decreasing**?
 - b. Which function is linear? _____
 - c. Which function is concave down? _____
 - d. Use the central difference quotient to determine $f'(2)$ and $f''(1.5)$
3. The cost C (in dollars) to produce g gallons of chocolate chunk ice cream can be expressed as $C=f(g)$.
- a. Is $f(g)$ more likely to be an increasing function or a decreasing function?
 - b. Explain the meaning of $f'(200)=1.6$ in terms of ice cream production. Be sure to include units!
4. Let $f(x) = 3\cos(4x - \pi)$
- a. $f(x)$ has amplitude _____ and period _____.
 - b. Is $f(x)$ EVEN, ODD, or NEITHER? Why?
 - c. Determine $f'(x)$ and $f''(x)$
5. Use the graph of the function $f(x)$ below to answer the following questions. Assume that the entire graph is shown below.



- a. Identify the points where $f(x)=0$, $f'(x)=0$, $f''(x)=0$
 - b. Over what interval(s) is $f'(x)<0$? _____
 - c. Over what interval(s) is $f''(x)<0$? _____
 - d. Determine $\lim_{x \rightarrow 2} f(x)$, $\lim_{x \rightarrow 4} f(x)$, $\lim_{x \rightarrow \infty} f(x)$
6. Let $f(x) = 4x^2 + x^3 - x^4$
- a. Use the Intermediate Value Theorem to show that there must be a number c with $2 \leq c \leq 3$ such that $f(c)=0$.
 - b. Determine $f'(x)$ and $f''(x)$
7. Evaluate these limits and simplify your result. If the limit does not exist, write "Does Not Exist." Show your work!!!
- a. $\lim_{x \rightarrow \frac{3\pi}{2}} (6x \sin x) =$ _____

b. For $f(x) = \begin{cases} 3x-1 & x < 4 \\ 8 & x = 4 \\ -x^2+2 & x > 4 \end{cases}$ $\lim_{x \rightarrow 4} f(x) = \underline{\hspace{2cm}}$
 $\lim_{x \rightarrow 4^+} f(x) = \underline{\hspace{2cm}}$
 $\lim_{x \rightarrow 0} f(x) = \underline{\hspace{2cm}}$

c. $\lim_{x \rightarrow 0} \frac{\sin 3x}{x} = \underline{\hspace{2cm}}$

d. $\lim_{x \rightarrow -3^+} \frac{|x+3|}{x+3} = \underline{\hspace{2cm}}$

e. $\lim_{x \rightarrow \infty} \frac{x^2 - 7x + 1}{3x^2} = \underline{\hspace{2cm}}$

8. Sketch the graph of a function that is continuous at $x=3$, but not differentiable at $x=3$.

9. Determine the derivative of each of the following:

a. $f(x) = \arccos(3x-1)$ b. $f(x) = 5^x \ln x$ c. $f(x) = \frac{x^2-1}{x^2+1}$

10. Give the equation of the line tangent to $f(x) = 5e^{4x}$ at $x=0$. At $x=1$

11. Sketch the graph of a continuous function $g(x)$ such that

$$\begin{aligned} g' &< 0 & -4 < x < 8 \\ g' &> 0 & x < -4, x > 8 \\ g'' &< 0 & x < 2 \\ g'' &> 0 & x > 2 \end{aligned}$$

12. Consider the function $f(x) = x^3 - 2x$

- On what interval(s) is $f(x)$ both increasing and concave down?
- Give the absolute maximum and minimum points of $f(x)$ on the interval $0 \leq x \leq 2$.

13. Consider $y^2 - x^3y = 2$

- Find $\frac{dy}{dx}$.
- Find the equation of the line tangent to the curve at the point $(1, -1)$.

14. A rectangle is to be inscribed within a right triangle with a base of 3 and a height of 4. Give the dimensions of the rectangle with the maximum area.

15. A 10-ft ladder leans against a wall. The base of the ladder is moving outward at a speed of 1 ft per second when the top of the ladder is 8 ft from the floor. At that moment, how fast is the top of the ladder sliding down the wall?

16. Let $f(t)$ be the rate at which the world population is growing in year t , in billion people per year. Explain in words what $\int_{2000}^{2005} f(t)dt$ represents. Give units.

17. Evaluate the indefinite integrals. Simplify.

a. $\int \left(1 + \frac{1}{x} + \frac{1}{\sqrt{x}} \right) dx =$ b. $\int \frac{6}{1+(3x)^2} dx =$ c. $\int \frac{\ln \sqrt{x}}{x} dx =$

18. Evaluate the definite integrals. Simplify.

a. $\int_1^2 3^x dx =$ b. $\int_1^3 \frac{4}{5x} dx =$ c. $\int_0^1 \frac{e^t + 1}{e^t + t} dt =$

19. Give an exact value for the area under $y=e^x$ and above $y=1$ for $0 \leq x \leq 2$.

20. Give an exact value for the area between the graphs of $f(x) = \sin(\pi x) + 4$ and $g(x) = 9 - 3x^2$ for $0 \leq x \leq 1$.