MAT220 Final Exam Review - SOLUTIONS

1. Let f(x) be an even function and g(x) be an odd function. Complete the following table.

x	*f(x)*	*g(x)*
-2	5	5
-1	4	4
0	0	0
1	4	-4
2	5	-5

2. a. decreasing b. g(x) is linear c. f(x) is concave down d. f'(2)= -4, f''(1.5)= -2

3. a. increasing b. When 200 gallons of ice cream are produced, the production cost is increasing at a rate of 1.6 dollars per gallon.

4. a. *f(x)* has amplitude 3 and period π/2. b. Even
   c. $f'(x) = -12\sin(4x-\pi)$ $f''(x) = -48\cos(4x-\pi)$

5. a. f(x)=0 when x=0, 4 f'(x)=0 when x=0, 2, 4 f''(x)=0 when x=1, 3
   b. *f'(x)*<0 when x<0, 2<x<4 c. *f''(x)*<0 when 1<x<3
   d. $\lim_{x\to 2} f(x) = 16,$ $\lim_{x\to 4} f(x) = 0,$ $\lim_{x\to\infty} f(x) = IWOB$

6. Let $f(x) = 4x^2 + x^3 - x^4$
   a. f(2) = 8 > 0 f(3) = -18 < 0, therefore, by the Intermediate Value Theorem, there must be a number *c* with 2 ≤ *c* ≤ 3 such that *f(c)=0*.
   b. $f'(x) = 8x + 3x^2 - 4x^3$ $f''(x) = 8 + 6x - 12x^2$

7. Evaluate these limits and simplify your result. If the limit does not exist, write "Does Not Exist." Show your work!!!
   a. $\lim_{x\to \frac{3\pi}{2}} (6x\sin x) = -9\pi$

      $\lim_{x\to 4} f(x) = DNE$
   b. $\lim_{x\to 4^+} f(x) = -14$

      $\lim_{x\to 0} f(x) = -1$
   c. $\lim_{x\to 0} \frac{\sin 3x}{x} = 3$
   d. $\lim_{x\to -3^+} \frac{|x+3|}{x+3} = 1$
   e. $\lim_{x\to\infty} \frac{x^2 - 7x + 1}{3x^2} = 1/3$

8. The graph must have a sharp point <u>or</u> a vertical tangent at x=3.

9. Determine the derivative of each of the following:

a. $f'(x)=\dfrac{-3}{\sqrt{1-(3x-1)^2}}$ b. $f'(x)=5^x\ln 5\ln x+\dfrac{5^x}{x}$ c. $f'(x)=\dfrac{4x}{\left(x^2+1\right)^2}$

10. At x=0: $y = 20x + 5$. At x=1: $y = 20e^4x - 15e^4$

11. Sketch the graph of a continuous function g(x) such that

$g'<0 \quad -4<x<8$

$g'>0 \quad x<-4, x>8$

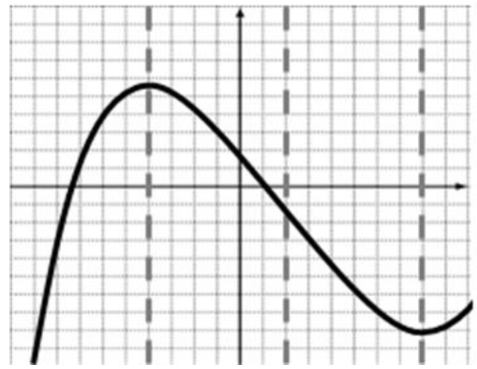
$g''<0 \quad x<2$

$g''>0 \quad x>2$

Local Max: x = -4

Local Min: x = 8

Point of Inflection: x = 2



12. a. $x<-\sqrt{\dfrac{2}{3}}$, $\left(-\infty,-\sqrt{\dfrac{2}{3}}\right)$

b. Absolute maximum at x=2, absolute minimum at x=$\sqrt{\dfrac{2}{3}}$

13. a. $\dfrac{dy}{dx}=\dfrac{3x^2y}{2y-x^3}$ b. $y = x - 2$

14. The rectangle with the maximum area: base= 1.5 units height= 2 units

15. -3/4 feet per second. (The answer is negative because the ladder is moving downward.)

16. $\int_{2000}^{2005} f(t)dt$ would give the *change in* population (in billions of people) from 2000 to 2005. For example, $\int_{2000}^{2005} f(t)dt$ =350 would mean that the population increased by 350 billion people during this time period.

17. Evaluate the indefinite integrals. Simplify.

a. $\int\left(1+\dfrac{1}{x}+\dfrac{1}{\sqrt{x}}\right)dx = x+\ln|x|+2\sqrt{x}+C$

b. $\int \dfrac{6}{1+(3x)^2}\,dx = 2\arctan(3x)+C$

c. $\int \dfrac{\ln\sqrt{x}}{x}\,dx=\left(\ln\sqrt{x}\right)^2+C$

18. Evaluate the definite integrals. Simplify.

a. $\int_1^2 3^x\,dx=\dfrac{6}{\ln 3}$ b. $\int_1^3 \dfrac{4}{5x}\,dx=\dfrac{4}{5}\ln 3$

c. $\int_0^1 \dfrac{e^t+1}{e^t+t}\,dt=\ln(e+1)$

19. $\int_0^2 e^x-1\,dx=e^2-3$

20. $\int_0^1\left[5-3x^2-\sin(\pi x)\right]dx=4-\dfrac{2}{\pi}$